

Extreme multistability in a memristive circuit

Bo-Cheng Bao[✉], Quan Xu, Han Bao and Mo Chen

A memristive Chua's circuit regarded as a paradigm is reconsidered to exhibit its extreme multistability in this Letter. Memristor initial state-dependent dynamics is analysed and the coexistence of infinitely many attractors related to memristor initial states is revealed by numerical simulations and circuit simulations. The dynamical behaviour just reflects the emergence of extreme multistability in the memristive circuit.

Introduction: Because of non-linearity of memristor, memristor based circuit or memristive circuit is a kind of special non-linear dynamical circuit [1]. It is reported in [2] that the dynamical stability of memristor based chaotic circuit is closely dependent on memristor initial state, which implies that there are infinitely many attractors in the memristive circuit. In the coupled Rössler oscillator with partial synchronisation, the phenomenon depending on the initial state of a certain variable is called extreme multistability [3, 4]. Thus, infinitely many attractors' behaviour closely relying on memristor initial state just reveals the emergence of extreme multistability in a memristive circuit.

In the past few years, realised by memristors to replace the non-linear resistors in some classical chaotic circuits, many memristive circuits with a line equilibrium have been presented [5, 6]. Taking a memristive Chua's circuit [6] as a paradigm in the present Letter, the long term dynamical behaviour associated with memristor initial state indicates that extreme multistability indeed exists in a memristive circuit.

Memristive circuit: Take a memristive Chua's circuit shown in Fig. 1a [6] as a paradigm. The flux-controlled memristor can be realised by discrete components, which is depicted in Fig. 1b [6]. The voltage-current relation of the memristor can be expressed as

$$i_M = W(v_0)v_M = (-a + b|v_0|)v_M \quad (1)$$

where $a, b > 0$, and $W(v_0)$ is the memductance function. The flux $\varphi_M(t)$ of the memristor is then given by

$$\varphi_M(t) = \int_{-\infty}^t v_M(\tau) d\tau = -R_0 C_0 v_0(t) \quad (2)$$



$R = 2 \text{ k}\Omega$, $C_1 = 6.8 \text{ nF}$, $C_2 = 68 \text{ nF}$, $L = 17.2 \text{ mH}$, $R_0 = 2 \text{ k}\Omega$, and $C_0 = 68 \text{ nF}$, and the non-linear element parameters are calculated as $a = 0.6667 \text{ mS}$ and $b = 0.403 \text{ mS/Wb}$.

Memristor initial state-dependent dynamics: Obviously, model (3) has a line equilibrium, which can be expressed by

$$S = \{(v_1, v_2, i_3, v_0) | v_1 = v_2 = 0 \text{ V}, i_3 = 0 \text{ A}, v_0 = c \text{ V}\} \quad (4)$$

where c is uncertain but constant. At the line equilibrium S , the characteristic polynomial is formulated as

$$P(\lambda) = \lambda(\lambda^3 + d_2\lambda^2 + d_1\lambda + d_0) = 0 \quad (5)$$

where

$$d_2 = \frac{1}{RC_1} + \frac{1}{RC_2} + \frac{a+b|c|}{C_1}$$

$$d_1 = \frac{a+b|c|}{RC_1C_2} + \frac{1}{LC_2}$$

$$d_0 = \frac{1}{RLC_1C_2} + \frac{a+b|c|}{LC_1C_2}$$

By Routh–Hurwitz conditions, the roots of the cubic polynomial inside brackets of (5) have positive real parts when and only when

$$|c| < 0.4136 \text{ V and } 0.5005 \text{ V} < |c| < 1.3658 \text{ V} \quad (6)$$

For any c in the region of (6), the memristive circuit modelled by (3) is unstable, which implies that the circuit dynamics is mainly affected by the initial state $v_0(0)$, i.e. memristor initial state-dependent dynamics exists in the memristive circuit.

Extreme multistability behaviour: Considering the circuit initial states as $v_1(0) = 1 \text{ nV}$, $v_2(0) = 0 \text{ V}$, and $i_3(0) = 0 \text{ A}$, when $v_0(0)$ is gradually increased from -1.5 to 1.5 V , the bifurcation diagram and the first three Lyapunov exponents are numerically shown in Fig. 2, from which the dynamical behaviours of point, periodicity, and chaos are observed. The results illustrate that the long term dynamical behaviour extremely depends on the memristor initial state thus leading to the emergence of extreme multistability in the memristive circuit [3, 4].



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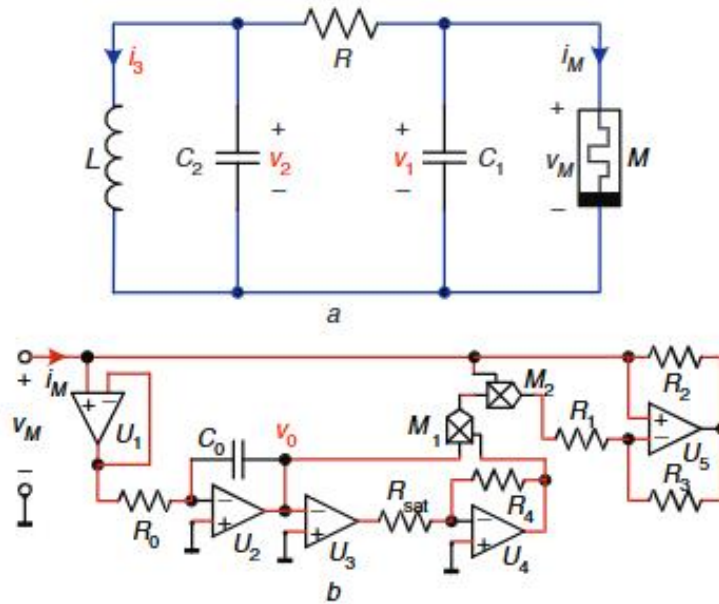


Fig. 1 Memristive circuit for illustration of extreme multistability

a Circuit schematic of memristive Chua's circuit
b Memristor equivalent circuit realisation

For the four circuit state variables of v_1 , v_2 , and i_3 , and v_0 in Fig. 1, the state equations are written as [6]

$$\begin{aligned} C_1 dv_1/dt &= (v_2 - v_1)/R - W(v_0)v_1 \\ C_2 dv_2/dt &= (v_1 - v_2)/R - i_3 \\ L di_3/dt &= v_2 \\ C_0 dv_0/dt &= -v_1/R_0 \end{aligned} \quad (3)$$

According to Bao *et al.* [6], the linear element parameters are set to

extremely depends on the memristor initial state thus leading to the emergence of extreme multistability in the memristive circuit [3, 4].

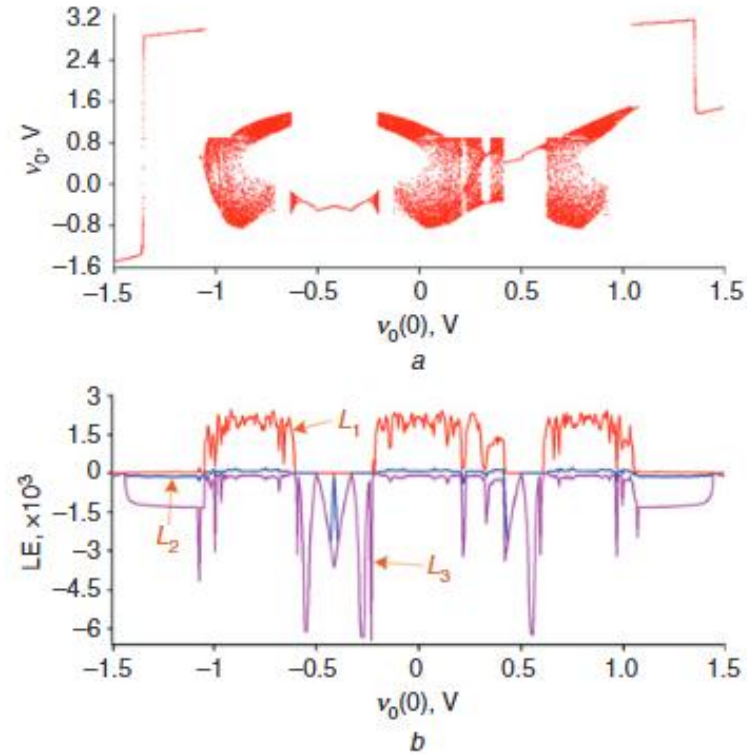


Fig. 2 Extreme multistability relying on memristor initial state $v_0(0)$

a Bifurcation diagram of $v_0(t)$
b Lyapunov exponent spectra

Infinitely many attractors: With model (3) and circuit schematic in Fig. 1, infinitely many attractors in the memristive circuit can be captured by MATLAB numerical simulations and PSIM circuit simulations.

For the above determined circuit parameters and initial states, when the memristor initial state $v_0(0)$ is set to different voltage values, the phase portraits of double-scroll chaotic attractors, upper and lower chaotic spiral attractors with different topologies, upper and lower limit cycles with different topologies and periodicities, and point

attractors on different locations are plotted in Fig. 3. Much more simulations demonstrate that infinitely many attractors can indeed be emerged in the memristive circuit.

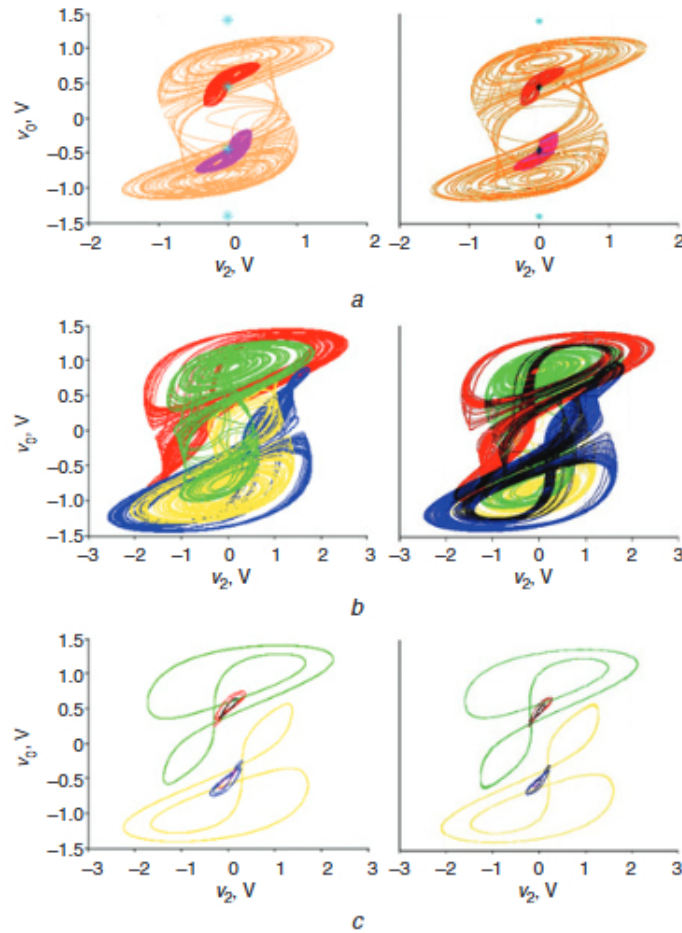


Fig. 3 Projections of infinitely many attractors on the v_2 - v_0 plane, where the lefts are MATLAB numerical simulations and the rights are PSIM circuit simulations

a $v_0(0) = 0$ V (orange), double-scroll chaotic attractor; $v_0(0) = 0.62$ V (red) and -0.21 V (magenta), upper and lower chaotic spiral attractors, respectively; $v_0(0) = -1.4$ V, -0.45 V, 0.45 V, 1.4 V (cyan), point attractors (marked with bold asterisks)
b $v_0(0) = 0.4$ V (blue) and 0.7 V (yellow), lower chaotic spiral attractors; $v_0(0) = 0.9$ V (green) and 1 V (red), upper chaotic spiral attractors
c $v_0(0) = -0.6$ V (blue) and -0.55 V (magenta), lower limit cycles; $v_0(0) = 0.33$ V (yellow), lower limit cycle; $v_0(0) = 0.6$ V (red) and 0.55 V (black), upper limit cycles; $v_0(0) = 0.995$ V (green), upper limit cycle

References

- 1 Chua, L.O.: 'The fourth element', *Proc. IEEE*, 2012, **100**, (6), pp. 1920–1927
- 2 Bao, B.C., Xu, J.P., and Liu, Z.: 'Initial state dependent dynamical behaviours in memristor based chaotic circuit', *Chin. Phys. Lett.*, 2010, **27**, (7), pp. 070504-1–070504-3
- 3 Hens, C., Dana, S.K., and Feudel, U.: 'Extreme multistability: attractors manipulation and robustness', *Chaos*, 2015, **25**, (5), pp. 053112-1–053112-7
- 4 Patel, M.S., Patel, U., Sen, A., Sethia, G.C., Hens, C., and Dana, S.K.: 'Experimental observation of extreme multistability in an electronic system of two coupled Rössler oscillators', *Phys. Rev. E*, 2014, **89**, (2), pp. 022918-1–022918-4
- 5 Kengne, J., Tabekoueng, Z.N., Tamba, V.K., and Nougou, A.N.: 'Periodicity, chaos, and multiple attractors in a memristor-based Shinriki's circuit', *Chaos*, 2015, **25**, (10), pp. 103126-1–103126-10
- 6 Bao, B.C., Hu, F.W., Liu, Z., and Xu, J.P.: 'Mapping equivalent approach to analysis and realization of memristor based dynamical circuit', *Chin. Phys. B*, 2014, **23**, (7), pp. 070503-1–070503-8

Conclusions: In this Letter, a memristive Chua's circuit taken as a paradigm is reconsidered, from which the phenomenon of extreme multistability revealing the coexistence of infinitely many attractors is exhibited. The results verified by PSIM circuit simulations demonstrate that the emergence of extreme multistability is initiated by memristor initial state-dependent dynamics in the memristive circuit. Considering the hard assignment of memristor initial state with different desired values, the striking dynamical behaviour of infinitely many attractors is difficultly captured in hardware experiments. To solve this question could be subject of further research.

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One or more of the Figures in this Letter are available in colour online.

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